Analysis of Flour Type, Butter Consistency, and Leavening Agents on the Rise of Vanilla Cupcakes

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Introduction:

The last several times that I made vanilla cupcakes, I have been suspicious that my gluten free cupcakes were not rising as much as their all-purpose flour cupcakes counterparts. I have tried several different recipes with different leavening agents and methods, which has led me to be interested in more formally testing this suspicion. This report aims to formally investigate the impact of different factors on the rise of vanilla cupcakes, with a particular focus on the effects of all-purpose flour vs. gluten free flour, the use of baking powder vs. baking soda, and the use of melted butter vs. room-temperature butter.

This analysis will test if there is a difference in cupcake rise between the gluten free flour and the all-purpose flour cupcakes, if there is a difference in rise between baking powder vs. baking soda cupcakes, and if there is a difference in rise between the melted butter vs. room-temperature butter cupcakes. Mean comparisons of any significant factors will be performed to assess what factors lead to the best rise in cupcakes.

Data:

The data were produced from an experiment that I designed and executed. The response variable is the height of the cupcakes in millimeters, and the factors are flour type (all-purpose vs. gluten free), butter consistency when added to the mix (melted vs. room-temperature), and leavening agent (baking powder vs. baking soda). All factors are fixed factors.

The treatment design is a 2x2x2 factorial design resulting in 8 treatment combinations. The experimental unit is a batch of cupcake mix, and each batch produces three cupcakes. The height of the three cupcakes from each batch was averaged to obtain 8 total observations. A completely randomized design was utilized to randomly assign the treatments to the batches.

SAS was used for both the randomization of the treatments to the batches, as well as the randomization of the order in which to bake the batches. The same cupcake recipe[1] was used for all 8 batches, with the three factors of interest being the only elements that varied between batches. The batches were all independently mixed, weighed into muffin tins, baked, and then the cupcakes were all allowed to cool completely. All other conditions, such as oven temperature, mixing tools, and the placement of the cupcake rack in the oven, were held constant across batches. When the cupcakes were cool, the height of the cupcakes at their center was

References: [1] https://www.justsotasty.com/small-batch-vanilla-cupcakes/#recipe

recorded using a toothpick. The heights of the three cupcakes from each batch were then averaged to obtain the final response value.

The distribution of the response for each of the three factors and their levels can be seen in Figures 1 through 3. There is a slight skew to the data when looking at the rise of the cupcakes split out by each of the factor levels. We can also see there are no missing values and no outliers in the data. No definitive conclusions can be made based on the boxplots; however, it appears that all-purpose flour, melted butter, and baking soda lead to a larger height when compared to gluten free flour, room temperature butter, and baking powder. Additional summary statistics of the data that were analyzed can be found in Tables A1 through A3 in Appendix A.

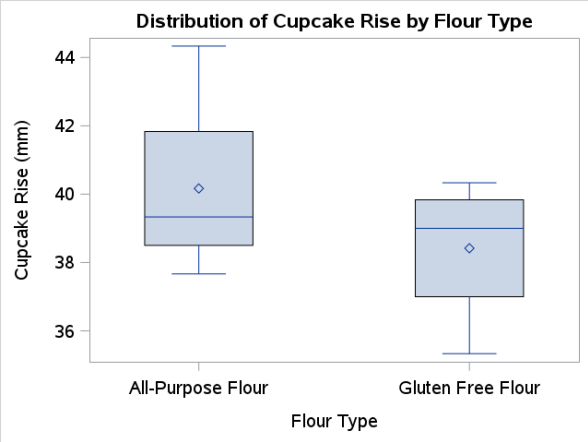

Figure 1: Height Distribution by Flour Type

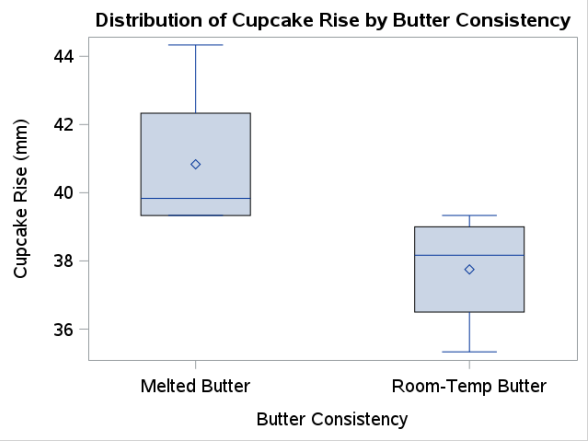

Figure 2: Height Distribution by Butter Consistency

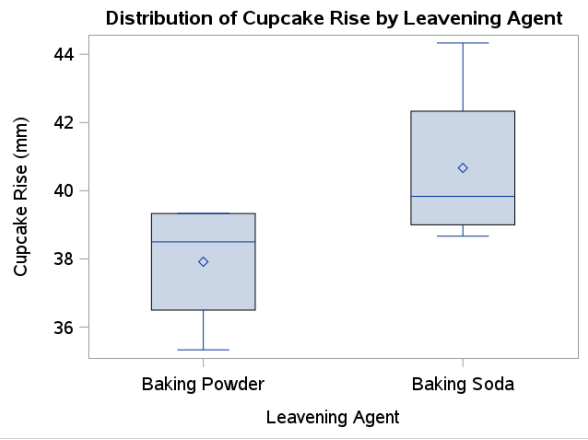

Figure 3: Height Distribution by Leavening Agent

Methods:

SAS was used to conduct all statistical analyses within this report. The full model considered included the factors flour type, butter consistency, and leavening agents, as well as the two-way interaction terms for the three factors. The three-way interaction term was not included because of the small dataset size and that the error degrees of freedom was 0 when included. At a 10% level of significance, all the two-way interaction terms were not significant

(p-values of 0.89, 0.75, and 0.89. See Table A4 in Appendix A for more details), and the model was reduced to only include the main effects. This resulted in the reduced statistical model:

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \epsilon_{ijk}$$

where y_{ijk} is the average rise of the cupcake for the kth leavening agent (k=baking powder, baking soda), jth butter consistency (j=melted, room-temp), and ith flour type (i=all-purpose, gluten free), μ is the overall mean, and $\epsilon_{ijk} \overset{iid}{\sim} N(0, \sigma^2)$. α_i is the effect of the ith flour type ($\sum_i \alpha_i = 0$). β_j is the effect of the jth butter consistency($\sum_j \beta_j = 0$). γ_k is the effect of the kth leavening agent ($\sum_k \gamma_k = 0$). Mean comparisons between the main effects using this model were assessed using the Tukey adjustment for pairwise comparisons using a 10% level of significance.

The studentized residuals were used to verify the ANOVA model assumptions. The Anderson Darling test affirmed normality of the residuals (p-value>0.25), and the errors are assumed to be independent from one another based on the experiment design. There was a slight pattern observed in the residuals when plotted against the predicted mean values which raised some concerns regarding the assumption of equal variances. However, given the extremely small sample size and the robustness of the ANOVA model, we proceeded with the model without transformation and take caution when it comes to interpreting the results. The output of the residual plots and normality tests can be found in Figure A1 and Table A5 in Appendix A.

Results:

Three separate hypotheses were tested using the final statistical model. The first was if there was a difference in the height of cupcakes due to the flour type used, the second was if there was a difference in the height of cupcakes due to the butter consistency used, and lastly if there is a difference in the height of cupcakes due to the leavening agent used.

The analysis yielded significant results for the flour type, butter consistency, and leavening agent effects. The F-statistics for these three main effects were 4.96, 15.38, and 12.24 respectively, all with 1 numerator degree of freedom and 4 denominator degrees of freedom. The p-values for these corresponding F-statistics are 0.09, 0.0172, and 0.0249. Therefore, at a 10% level of significance, we reject the null hypotheses and conclude all three of these factors significantly influence the height of cupcakes. A full output of the ANOVA table can be found in Table A6 in Appendix A.

Since all three of the main effects were found to be significant, comparison of means was completed to assess what factors led to a larger rise in cupcakes. As seen in Figures 4 through 6,

the all-purpose flour led to a statically higher rise in the cupcakes when compared to the gluten free flour, the melted butter led to a statically higher rise in the cupcakes when compared to the room-temperature butter, and finally the baking soda led to a statically higher rise in the cupcakes when compared to the baking powder.

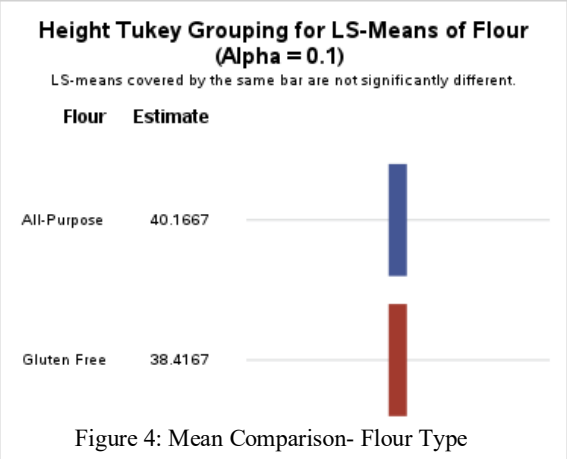

Figure 4: Mean Comparison- Flour Type

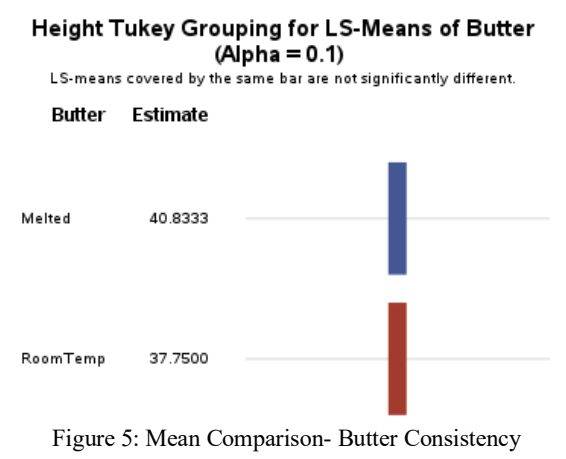

Figure 5: Mean Comparison- Butter Consistency

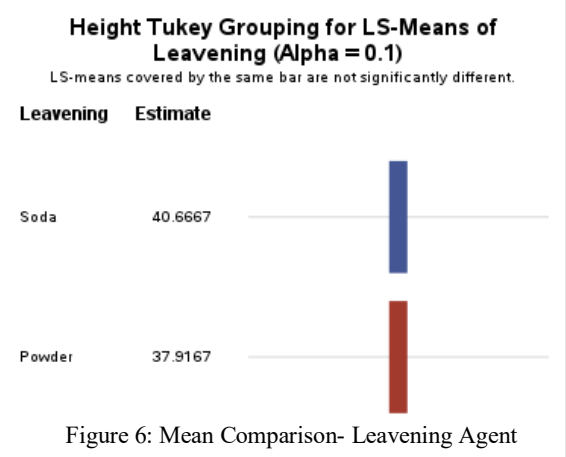

Figure 6: Mean Comparison- Leavening Agent

Conclusion:

This analysis investigated the impact of flour type, butter consistency, and leavening agent on the rise of vanilla cupcakes. All three of these factors were found to significantly influence the height of the cupcakes. Specifically, all-purpose flour, melted butter, and baking soda led to the cupcakes with the highest rise.

Due to the small sample size, certain limitations were encountered during the analysis. First, it was not possible to include a three-way interaction term in the initial model, and there was reduced statistical power to detect effects. Second, the small sample size led to difficultly assessing if the model violated the ANOVA assumptions, leading caution needed when interpreting the results. To address these limitations, it is recommended to replicate this experiment with a larger sample size and re-run the analysis.

Appendix A:

Table A1: Summary Statistics for the Flour Type Factor

Flour Type	N	Mean	Std Dev	Minimum	Maximum
All-Purpose	4	40.17	2.89	37.67	44.33
Gluten Free	4	38.42	2.17	35.33	40.33

Table A2: Summary Statistics for the Butter Consistency Factor

Butter Consistency	N	Mean	Std Dev	Minimum	Maximum
Melted Butter	4	40.83	2.38	39.33	44.33
Room-Temperature Butter	4	37.75	1.75	35.33	39.33

Table A3: Summary Statistics for the Leavening Agent Factor

Leavening Agent	N	Mean	Std Dev	Minimum	Maximum
Baking Powder	4	37.92	1.89	35.33	39.33
Baking Soda	4	40.67	2.54	38.67	44.33

Table A4: ANOVA Analysis Output of the Full Model Including All Main Effects and Two-way Interaction Terms

Type 3 Analysis of Variance								
Source	**DF**	**Sum of Squares**	**Mean Square**	**Expected Mean Square**	**Error Term**	**Error DF**	**F Value**	**Pr > F**
Flour	1	6.125000	6.125000	Var(Residual) + Q(Flour,Flour*Butter,Flour*Leavening)	MS(Residual)	1	1.53	0.4332
Butter	1	19.013889	19.013889	Var(Residual) + Q(Butter,Flour*Butter,Butter*Leavening)	MS(Residual)	1	4.74	0.2742
Leavening	1	15.125000	15.125000	Var(Residual) + Q(Leavening,Flour*Leavening,Butter*Leavening)	MS(Residual)	1	3.77	0.3028
Flour*Butter	1	0.125000	0.125000	Var(Residual) + Q(Flour*Butter)	MS(Residual)	1	0.03	0.8888
Flour*Leavening	1	0.680556	0.680556	Var(Residual) + Q(Flour*Leavening)	MS(Residual)	1	0.17	0.7513
Butter*Leavening	1	0.125000	0.125000	Var(Residual) + Q(Butter*Leavening)	MS(Residual)	1	0.03	0.8888
Residual	1	4.013889	4.013889	Var(Residual)	.	.	.	.

Table A5: Output from the Test of Error Normality

Tests for Normality				
Test	**Statistic**		**p Value**	
Shapiro-Wilk	**W**	0.924108	**Pr < W**	0.4641
Kolmogorov-Smirnov	**D**	0.189971	**Pr > D**	>0.1500
Cramer-von Mises	**W-Sq**	0.044645	**Pr > W-Sq**	>0.2500
Anderson-Darling	**A-Sq**	0.29261	**Pr > A-Sq**	>0.2500

Figure A1: Output of the Studentized Residuals using the Reduced ANOVA Model

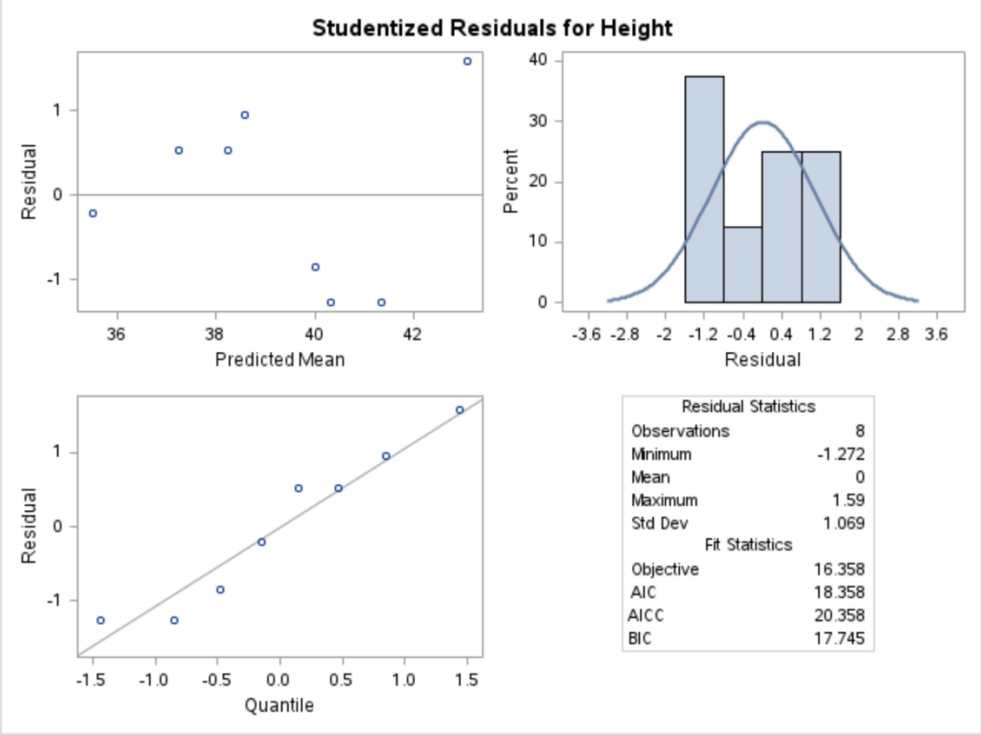

Table A6: ANOVA Analysis Output of the Reduced Final Model

Type 3 Analysis of Variance								
Source	**DF**	**Sum of Squares**	**Mean Square**	**Expected Mean Square**	**Error Term**	**Error DF**	**F Value**	**Pr > F**
Flour	1	6.125000	6.125000	Var(Residual) + Q(Flour)	MS(Residual)	4	4.96	0.0900
Butter	1	19.013889	19.013889	Var(Residual) + Q(Butter)	MS(Residual)	4	15.38	0.0172
Leavening	1	15.125000	15.125000	Var(Residual) + Q(Leavening)	MS(Residual)	4	12.24	0.0249
Residual	4	4.944444	1.236111	Var(Residual)	.	.	.	.